1 Optimization

First we focus on the one dimensional, unconstrained cases, and discuss the finding of the root of $g'(x)$ where $g(x)$ is our objective function. The problem boils down to the finding of the root of g'.

- Bisection search: can find only one of the roots if there are more than one; need initialize the algorithm properly such that $g'(a)g'(b) < 0$ where a and b are two end points of the search.
- Newton-Raphson method: use Taylor expansion of g' at the current solution $x^{(t)}$: $g'(x^*) \approx g'(x^{(t)}) + g''(x^{(t)})(x^* - x^{(t)})$. Then let $g'(x^*) = 0$, $x^* = x^{(t)} - g'(x^{(t)})/g''(x^{(t)})$. g needs to be second order differentiable and $g'' \neq 0$.
- Fisher scoring: mainly used in MLE where one wants to maximize the log-likelihood ℓ. In the Newton-Raphson, replace $-g''$ by the Fisher information $\mathrm{I} = \mathrm{E}(-\ell'')$ and we get: $x^* = x^{(t)} + \ell'(x^{(t)})/\mathrm{E}(-\ell''(x^{(t)}))$.
- Secant method: replace g'' by the approximation $(g'(x^{(t)}) - g'(x^{t-1}))/(x^{(t)} - x^{(t-1)})$.
- Fixed point iteration
- Golden section search

$$ \tag{1} $$

2 EM algorithm