Homework #6: Chapters 14-15

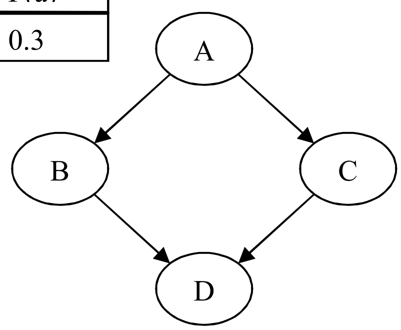
The following exercises are due at the beginning of class on Tuesday, April 25. Note, this homework is continued on the reverse side of the paper.

1. *[20 pts.]* Do exercise 14.11 (a-d) from the book (p. 561).

2. *[25 pts.]* Consider the Bayesian network below, where A, B, C, and D are all Boolean random variables.

P(a)
0.3

A	P(b|A)
T	0.7
F	0.3



A	P(c|A)
T	0.5
F	0.2

B	C	P(d|B,C)
T	T	0.4
T	F	0.1
F	T	0.5
F	F	0.9

Compute the following probabilities and probability distributions, using a <true,false> ordering for all Boolean variable probability distributions. You must give computed numeric answers and show all of your work.

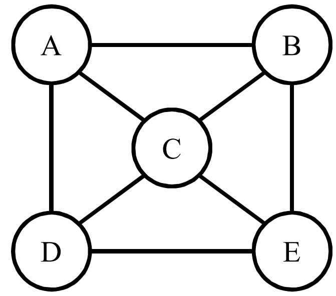
a) $P(a \wedge b \wedge \neg c \wedge d)$
b) $\mathbf{P}(A \mid b \wedge c \wedge \neg d)$
c) $\mathbf{P}(B \mid a \wedge d)$

3. *[20 pts.]* We have a bag of three biased coins *a*, *b*, and *c* with probabilities of coming up heads of 30%, 40% and 90%, respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X_1, X_2, and X_3.

   a) Draw the Bayesian network corresponding to this setup and define the necessary conditional probability tables (CPTs).

   b) Calculate which coin was most likely to have been drawn from the bag if the observed flips come out heads twice and tails once (in some order).

4. *[25 pts.]* Consider a computer virus moving between nodes in the given network. At each time step, the virus moves between nodes according to the following rules: it has a 16% chance of remaining at the same node, otherwise it is equally likely to move to any one of the connected nodes. We do not know exactly where on the network the virus is, but at each time step a virus scanner reports its estimated position: each node connected to the virus's node has a 5% chance of being reported, and each non-connected node (other than the correct one) has a 2% chance of being reported, otherwise the estimated position is correct. Give a *Bayesian* network structure (similar to the one in Figure 15.2, p. 569) for this Markov process, including values for all relevant conditional probability tables. Use two discrete variables $Virus_t$ and $Scan_t$ to represent each state. The domain of both of these variables is the five nodes: A, B, C, D, or E.



5. *[10 pts.]* Show that any second-order Markov process can be rewritten as a first-order Markov process with an augmented set of state variables. Can this always be done without increasing the number of parameters needed to specify the transition model, i.e., *parsimoniously*?