Homework #7: Chapters 18

1. [10 pts.] Note the sport does not need to be tennis! The key things in that the answer makes correct analogies to the performance element, the learning element, the critic, and the problem generator, and discusses whether supervised learning, reinforcement learning or both occur. Here’s a sample answer...

Tennis: The primary sensors are the eye, although ears and touch can also play a role. The actuators are muscles, particularly in order to move to different positions on the court and to swing at the ball. The performance element must decide where to move, when to swing, and what kind of swing to use based on the percepts. We can think of three things that must be learned: movement, playing strokes, and strategy. The critic is the player’s self-evaluation of progress. It will consider whether the player has won the most recent point, a game, set or match. Based on this, the learning element makes changes to the performance element, ideally improving movement and swings. The problem generator leads to some experimenting on the players part: what happens if I alter my swing like this? What if I fake like I’m going to smash the ball and then gently drop it near the net?

The learning process involves both supervised and reinforcement learning. Supervised learning occurs as the agent works with an instructor that explains what to do in certain situations, and also as the agent develops predictive models of the environment: e.g. where will the ball land when someone hits it a certain way. When a point is won or lost, then reinforcement learning comes into play. Note, the reinforcement is not immediate with respect to each action, but only comes after a series of actions. Therefore, the learning must assess which actions deserve the most credit/blame when updating the performance element.
2. [30 pts.] Allow for some rounding errors, although all answers should be significant to at least two decimal places.

a. [4 pts] \( I\left(\frac{3}{8},\frac{5}{8}\right) = -\frac{3}{8}\log_2\frac{3}{8} - \frac{5}{8}\log_2\frac{5}{8} \approx 0.954 \)

b. [16 pts., 4pts. each calculation]
\[ \text{Gain(Color)} \]
\[ = 0.954 - \left[ \frac{1+2}{8} I\left(\frac{1}{3},\frac{2}{3}\right) + \frac{0+1}{8} I\left(\frac{0}{1},\frac{1}{1}\right) + \frac{1+1}{8} I\left(\frac{1}{2},\frac{1}{2}\right) + \frac{1+1}{8} I\left(\frac{1}{2},\frac{1}{2}\right) \right] \]
\[ = 0.954 - [0.375(0.918) + 0.125(0) + 0.25(1) + 0.25(1)] \]
\[ = 0.954 - [0.344 + 0.25 + 0.25] = 0.954 - 0.844 = 0.110 \]

\[ \text{Gain(Legs)} = 0.954 - \left[ \frac{0+1}{8} I\left(\frac{0}{1},\frac{1}{1}\right) + \frac{0+3}{8} I\left(\frac{0}{3},\frac{3}{3}\right) + \frac{3+1}{8} I\left(\frac{3}{4},\frac{1}{4}\right) \right] \]
\[ = 0.954 - [0.125(0) + 0.375(0) + 0.5(0.811)] \]
\[ = 0.954 - 0.405 = 0.549 \]

\[ \text{Gain(Tail)} = 0.954 - \left[ \frac{0+2}{8} I\left(\frac{0}{2},\frac{2}{2}\right) + \frac{3+3}{8} I\left(\frac{3}{6},\frac{3}{6}\right) \right] \]
\[ = 0.954 - [0.25(0) + 0.75(1)] \]
\[ =0.954 - 0.75 = 0.204 \]

\[ \text{Gain(Fur)} = 0.954 - \left[ \frac{0+3}{8} I\left(\frac{0}{3},\frac{3}{3}\right) + \frac{3+2}{8} I\left(\frac{3}{5},\frac{2}{5}\right) \right] \]
\[ = 0.954 - [0.375(0) + 0.625(0.971)] \]
\[ = 0.954 - 0.607 = 0.347 \]

c. [10 pts.] 2If Fur is used at the second level based on the above calculations (to choose the next attribute, you would have to redo calculations (entropy and gain) for just the four example X1, X3, X5, and X8).

\[ \text{Legs?} \]
\[ 0 \quad 2 \\
\[ \begin{align*}
+: X1,X5,X8 \\
-: &X4 \\
\end{align*} \]
\[ 4 \quad ? \\
\[ \begin{align*}
+: &X2,X3,X4,X6,X7 \\
-: &X2,X6,X7 \\
\end{align*} \]
\[ \begin{align*}
+: &X1,X5,X8 \\
-: &X3 \\
\end{align*} \]
3. [25 pts.] One point per weight in each row. Partial credit if early error is sole cause of future errors.

<table>
<thead>
<tr>
<th>step</th>
<th>W0</th>
<th>X0</th>
<th>W1</th>
<th>X1</th>
<th>W2</th>
<th>X2</th>
<th>in</th>
<th>out</th>
<th>Y</th>
<th>Err</th>
<th>change in W0</th>
<th>change in W1</th>
<th>change in W2</th>
<th>changed?</th>
</tr>
</thead>
<tbody>
<tr>
<td>initial</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.2</td>
<td>-0.5</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>-1</td>
<td>-0.5</td>
<td>0</td>
<td>0.1</td>
<td>1</td>
<td>-0.1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-0.1</td>
<td>0</td>
<td>0</td>
<td>+0.1 Y</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>-1</td>
<td>-0.5</td>
<td>1</td>
<td>0.2</td>
<td>0</td>
<td>-0.6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0 N</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>-1</td>
<td>-0.5</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td>-0.1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-0.1</td>
<td>0</td>
<td>0</td>
<td>0 Y</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>-1</td>
<td>-0.5</td>
<td>1</td>
<td>0.2</td>
<td>1</td>
<td>-0.3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-0.1</td>
<td>+0.1</td>
<td>+0.1 Y</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.1</td>
<td>-1</td>
<td>-0.4</td>
<td>0</td>
<td>0.3</td>
<td>1</td>
<td>0.4</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0 N</td>
</tr>
<tr>
<td>6</td>
<td>-0.1</td>
<td>-1</td>
<td>-0.4</td>
<td>1</td>
<td>0.3</td>
<td>0</td>
<td>-0.3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0 N</td>
</tr>
<tr>
<td>7</td>
<td>-0.1</td>
<td>-1</td>
<td>-0.4</td>
<td>0</td>
<td>0.3</td>
<td>0</td>
<td>0.1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0 N</td>
</tr>
<tr>
<td>8</td>
<td>-0.1</td>
<td>-1</td>
<td>-0.4</td>
<td>1</td>
<td>0.3</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0 N</td>
</tr>
</tbody>
</table>

The final weights are:
\[ w_0 = -0.1 \]
\[ w_1 = -0.4 \]
\[ w_2 = 0.3 \]

4. [20 pts.] Three points each for the calculation of \( a_4 \) to \( a_9 \). -1 per node in last two layers if error is only due to incorrect input from previous layer.

To more clearly show the calculations, we generalize the threshold function to specify a threshold parameter: It is also okay to subtract the threshold from the input in the calculations.

Threshold function: \[ g(x, t) = \begin{cases} 
0, & \text{if } x < t \\
1, & \text{if } x \geq t 
\end{cases} \]

\[ a_4 = g(in_4, t_4) = g(a_1 \cdot w_{1,4} + a_2 \cdot w_{2,4}, t_4) \]
\[ = g(1 \cdot 5 + 1 \cdot 2, 1) = g(7, 1) \]
\[ = 1 \]

\[ a_5 = g(in_5, t_5) = g(a_1 \cdot w_{1,5} + a_2 \cdot w_{2,5} + a_3 \cdot w_{3,5}, t_5) \]
\[ = g(1 \cdot 3 + 1 \cdot 2 + 0 \cdot -3, 4) = g(5, 4) \]
\[ = 1 \]

\[ a_6 = g(in_6, t_6) = g(a_2 \cdot w_{2,6} + a_3 \cdot w_{3,6}, t_6) \]
\[ = g(1 \cdot 2 + 0 \cdot 3, 2) = g(2, 2) \]
\[ = 1 \]

\[ a_7 = g(in_7, t_7) = g(a_4 \cdot w_{4,7} + a_5 \cdot w_{5,7} + a_6 \cdot w_{6,7}, t_7) \]
\[ = g(1 \cdot 3 + 1 \cdot -3 + 1 \cdot 2, 4) = g(2, 4) \]
\[ = 0 \]

\[ a_8 = g(in_8, t_8) = g(a_4 \cdot w_{4,8} + a_5 \cdot w_{5,8} + a_6 \cdot w_{6,8}, t_8) \]
\[ = g(1 \cdot 4 + 1 \cdot 1 + 1 \cdot -5, 2) = g(0, 2) \]
\[ = 0 \]

\[ a_9 = g(in_9, t_9) = g(a_7 \cdot w_{7,9} + a_8 \cdot w_{8,9}, t_9) \]
\[ = g(0 \cdot -4 + 0 \cdot 4, -1) = g(0, -1) \]
\[ = 1 \]
5. [15 pts.] 8 pts. for calculating the new point coordinates (2 pts. per example). 3 pts. for drawing the graph. 2 pts for the equation for the maximum margin separator and 2 pts for the margin.

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$y$</th>
<th>$F_1(x_1,x_2)=x_1$</th>
<th>$F_2(x_1,x_2)=x_1x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Maximum margin separator: the x-axis, i.e., $f_2=0$ or $x_1x_2=0$

The margin is 1. All four points are distance 1 from the separator.