Homework #6: Chapters 14-16

1. [25 pts.]
   a) [4 pts.] $P(a \land \neg b \land c \land d) = P(a) \times P(\neg b \mid a) \times P(c \mid a) \times P(d \mid \neg b \land c) = 0.7 \times 0.7 \times 0.2 \times 0.9 = 0.0882$

   b) [6 pts.] $P(A \mid b \land \neg c \land \neg d) = \alpha P(A, b, c, \neg d) = \alpha <0.7, 0.3> \times <0.3, 0.8> \times <0.8, 0.7> \times 0.6 = \alpha <0.1008, 0.1008> \Rightarrow \alpha = 4.9603$
   So, $<0.5, 0.5>$

   c) [15 pts.] $P(B \mid c \land \neg d) = \alpha P(B, c, \neg d) = \alpha <P(b \land c \land \neg d), P(\neg b \land c \land \neg d)>
   P(b \land c \land \neg d) = \sum_{\hat{a}} P(\hat{a}) \times P(b \mid \hat{a}) \times P(c \mid \hat{a}) \times P(\neg d \mid b \land c)$
   $= P(\neg d \mid b \land c) \times \sum_{\hat{a}} P(\hat{a}) \times P(b \mid \hat{a}) \times P(c \mid \hat{a})$
   $= 0.1 \times (0.7 \times 0.7 \times 0.2 + 0.3 \times 0.2 \times 0.3) = 0.0116$

   $P(\neg b \land c \land \neg d) = \sum_{\hat{a}} P(\hat{a}) \times P(\neg b \mid \hat{a}) \times P(c \mid \hat{a}) \times P(\neg d \mid \neg b \land c)$
   $= 0.1 \times (0.7 \times 0.7 \times 0.2 + 0.3 \times 0.2 \times 0.3) = 0.0116$

   So, $P(B \mid \neg c \land d) = \alpha <0.1026, 0.0116> \Rightarrow \alpha = 8.7566$
   So, $<0.8984, 0.1016>$
2. [25 pts.]

a) [3 pts] Although (i) in some sense depicts the “flow of information” during calculation, it is clearly incorrect as a network, since it says that given the measurements M1 and M2, the number of stars is independent of the focus. (ii) correctly represents the causal structure: each measurement is influenced by the actual number of stars and the focus, and the two telescopes are independent of each other. (iii) shows a correct but more complicated network—the one obtained by ordering the nodes M1, M2, N, F1, F2. If you order M2 before M1 you would get the same network except with the arrow from M1 to M2 reversed.

b) [3 pts] (ii) requires few parameters, and is therefore better than (iii).

c) [15 pts] To solve this problem, first consider that M1 is conditionally independent of F2 and M2, given F1 and M1. Given this, the approach to exact inference for computing P(M1|N) can be simplified to:

\[ P(M_1|N) = \alpha \sum_{\tilde{f}_1} P(\tilde{f}_1, N, M_1) \]
\[ = \alpha \sum_{\tilde{f}_1} P(\tilde{f}_1)P(N)P(M_1|\tilde{f}_1, N) \]
\[ = \alpha P(N) \sum_{\tilde{f}_1} P(\tilde{f}_1)P(M_1|\tilde{f}_1, N) \]

but since \( \alpha = 1/P(N) \), this simplifies to

\[ = \sum_{\tilde{f}_1} P(\tilde{f}_1)P(M_1|\tilde{f}_1, N) \]
\[ = P(\tilde{f}_1)P(M_1|\tilde{f}_1, N) + P(\neg \tilde{f}_1)P(M_1|\neg \tilde{f}_1, N) \]
\[ = (f)P(M_1|\tilde{f}_1, N) + (1-f)P(M_1|\neg \tilde{f}_1, N) \]

For a given N, we can determine the probabilities of each M1, by considering what kind of errors (or non-errors) must occur to give that measurement. Note for N=1, M1=0, the (1-f)e indicates the probability that there was an off-by-one without an out-of-focus; the (1-f) is to avoid counting off-by-one errors that co-occur with out-of-focus events.

<table>
<thead>
<tr>
<th>N</th>
<th>M1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>f + (1-f)e</td>
</tr>
<tr>
<td>1</td>
<td>f</td>
</tr>
<tr>
<td>2</td>
<td>f</td>
</tr>
<tr>
<td>3</td>
<td>f</td>
</tr>
</tbody>
</table>

1 pt for each entry in the table.

d) [4 pts] We can determine the possible values of N that are consistent with each measurement:

N(M1) = \{0, 1, 2, 4+\}, N(M2) = \{2, 3, 4, 6+\}

The intersection of these two sets is the answer \{2, 4, 6+\}
3. [20 pts.] We define the state variable $\text{Enough}_t$ and two evidence variables: $\text{RedEyes}_t$ and $\text{SleepInClass}_t$.

$\mathbf{P}(\text{Enough}_0) = <0.7, 0.3>$

| $E_{t-1}$ | $P(E_t|E_{t-1})$ |
|-----------|------------------|
| $t$       | 0.8              |
| $f$       | 0.3              |

$\begin{array}{|c|c|} \hline 
E_t & P(R_t|E_t) \\
\hline 
t & 0.2 \\
\hline 
f & 0.7 \\
\hline 
\end{array}$

$\begin{array}{|c|c|} \hline 
S_t & P(S_t|E_t) \\
\hline 
t & 0.1 \\
\hline 
f & 0.3 \\
\hline 
\end{array}$

2 pts for each of the four nodes (including proper links) Note, $\text{Enough}_{t+1}$ is optional
3 pts. for each of the conditional probability tables
3 pts for the prior of $\text{Enough}_0$
4. [15 pts.]
   a) [10 pts.] 1 pts. for each probability, 4 points for EMV calculation and answer
   P(BAR/BAR/BAR) = (0.25)(0.25)(0.25) = 0.015625
   P(BELL/BELL/BELL) = (0.25)(0.25)(0.25) = 0.015625
   P(LEMON/LEMON/LEMON) = (0.25)(0.25)(0.25) = 0.015625
   P(CHERRY/CHERRY/CHERRY) = (0.25)(0.25)(0.25) = 0.015625
   P(CHERRY/CHERRY/?) = (0.25)(0.25)(0.75) = 0.046875
   note the last bar can be anything but Cherry (otherwise it would be the prior payoff), thus .75
   P(CHERRY/?/?) = (0.25)(0.75)(1) = 0.1875
   note, a common mistake is to think that this is Cherry followed by two non-cherries = (0.25)(0.75)(0.75), but in fact the last bar could be Cherry, as long as the second wasn’t also Cherry
   EMV(pull) = 20(0.015625) + 15(0.015625) + 5(0.015625) + 3(0.015625) + 2(0.046875) + 1(0.1875)
               = 0.3125 + 0.234375 + 0.078125 + 0.046875 + 0.09375 + 0.1875 = 0.953125
   Thus the EMV < $1 (the cost of a pull). Therefore the game is not worth the price to play. Of course, if the EMV > $1, then the casino would never make money. Note, in addition to EMV not being the same as utility, some people get utility from the thrill of the possibility of a big win, and this is why slot machines are still popular, although in the long run you will lose money.

   b) [5 pts.] This is simply the sum of the probabilities from above: 0.296875.

5. [15 pts.] 4 points for each expected utility, and 3 points for final answer
   List of utility functions
   Utility(keep control) = 10
   Utility(pass to teammate) = 20
   Utility(score) = 100
   Utility(lose control to opponent) = -30

<table>
<thead>
<tr>
<th>Actions</th>
<th>keep control</th>
<th>pass to teammate</th>
<th>score</th>
<th>lose control</th>
</tr>
</thead>
<tbody>
<tr>
<td>dribble</td>
<td>0.8</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>pass</td>
<td>0</td>
<td>0.7</td>
<td>0</td>
<td>0.2 + 0.1 = 0.3</td>
</tr>
<tr>
<td>shoot</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>0.6 + 0.2 = 0.8</td>
</tr>
</tbody>
</table>

EU(dribble) = 0.8(10) + 0.2(-30) = 8 – 6 = 2
EU(pass) = 0.7(20) + (0.3)(-30) = 14 – 9 = 5
EU(shoot) = 0.2(100) + (0.8)(-30) = 20 -24 = -4

Maximum Utility(A) is when A = pass, so the robot should choose pass action.